ECE 333 Green Electric Energy

Homework 4 - Solution

P494-7.6:

a. the probability density function of wind speed is:

$$f(\underset{\sim}{V}=v)=\begin{cases}\frac{k}{10}v & 0\le v\le 10\\ 0 & otherwise\end{cases}$$

thus

$$\int_{-\infty}^{\infty} f(v)\,dv=\int_{0}^{10}\frac{k}{10}v\,dv=\frac{k}{20}(10^2-0^2)=5k=1 \qquad k=0.2$$

b.

$$E(\underset{\sim}{P})=E(\frac{1}{2}\rho A\underset{\sim}{V}^3)=\frac{1}{2}\rho A\cdot E(\underset{\sim}{V}^3)=\frac{1}{2}\rho A\cdot\int_{-\infty}^{\infty}v^3\cdot f(v)\,dv=\frac{1}{2}\rho A\cdot\int_{0}^{10}v^3\cdot 0.02v\,dv$$

$$=\frac{1}{2}1.225A\cdot\frac{0.02}{5}\cdot(10^5-0^5)$$

$$\frac{E(\underset{\sim}{P})}{A}=245\ \ W/m^2$$

P495-7.7:

For k = 2, the Weibull distribution is called the Rayleigh *p.d.f.*

$$f(v)=\frac{2v}{c^2}\,e^{-\left(\frac{v}{c}\right)^2}$$

$$\bar{v}\ =\ \int_0^{\infty} v\ f_{\underset{\sim}{V}}\ dv\ =\ 2\int_0^{\infty}\left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2}\ dv\ =\ \frac{\sqrt{\pi}}{2}c$$

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V}\le v\right)\Big|_{Rayleigh}\ =1-e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]}$$

Since the average wind speed is 9 *m/s*

$$c=\frac{2}{\sqrt{\pi}}\bar{v}\ =\frac{2}{\sqrt{\pi}}9=10.157$$

a. the probability that the wind speed is bigger than 25 *m/s*

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \geq 25\right)\Big|_{Rayleigh} = 1 - F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \leq 25\right)\Big|_{Rayleigh} = e^{-\left[\frac{\pi}{4}\left(\frac{25}{9}\right)^2\right]} = 2.34 \times 10^{-3}$$

the average hours over a year when the wind speed is bigger than 25 *m/s*

$$8760 \cdot F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \geq 25\right)\Big|_{Rayleigh} = 20.51 \ \ hours$$

thus there are 20.51 hours per year in average when the turbine will be shut down because of excessively high-speed wind

b. the probability that the wind speed is smaller than 5 *m/s*

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \leq 5\right)\Big|_{Rayleigh} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{5}{9}\right)^2\right]} = 0.215$$

the average hours over a year when the wind speed is smaller than 5 *m/s*

$$8760 \cdot F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \leq 5\right)\Big|_{Rayleigh} = 1884.86 \ \ hours$$

thus there are 1884.86 hours per year in average when the turbine will be shut down because the wind speed are too low

c. the probability that the wind speed is smaller than 25 *m/s* and bigger than 12 *m/s*

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \leq 25\right)\Big|_{Rayleigh} - F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \leq 12\right)\Big|_{Rayleigh} = e^{-\left[\frac{\pi}{4}\left(\frac{12}{9}\right)^2\right]} - e^{-\left[\frac{\pi}{4}\left(\frac{25}{9}\right)^2\right]} = 0.245$$

the average hours over a year when the wind speed is smaller than 25 *m/s* and bigger than 12 *m/s*

$$8760 \cdot 0.245 = 2149.30 \ \ hours$$

when the wind speed is smaller than (cut-off speed) 25 *m/s* and bigger than (rated wind speed) 12 *m/s,* the power output of the wind turbine is 1 *MW.* Thus the average energy produced by the wind at or above 12 *m/s*

$$2149.30 \ \ MWh$$

Problem a

(*i*)

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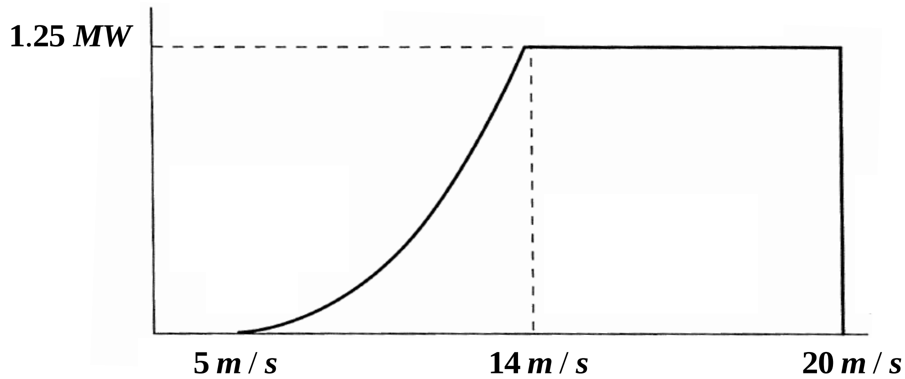

(*ii*) if the wind blows continuously between 15 and 20 m/s all day, the power output is 1.25 *MW* all day. Thus the total energy is 1.25×24=30 *MWh/day*

(*iii*) No.

Problem b

For k = 2, the Weibull distribution is called the Rayleigh *p.d.f.*

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}$$

$$\bar{v} = \int_0^\infty v\, f_{\underset{\sim}{V}}\, dv = 2\int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c$$

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \le v\right)\Big|_{Rayleigh} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]}$$

Since the average wind speed is 20 *m/s*

(*i*).

$$c = \frac{2}{\sqrt{\pi}}\bar{v} = \frac{2}{\sqrt{\pi}} 20 = 22.53$$

when temperature is 15 ^oC

$$\rho_{10m,15^oC} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15+15}\exp(-0.0342\frac{10}{273.15+15}) = 1.224\ kg/m^3$$

$$E(\underset{\sim}{p}) = E(\frac{1}{2}\rho \underset{\sim}{V}^3) = \frac{1}{2}1.224 \cdot E(\underset{\sim}{V}^3) \approx \frac{1}{2}1.224 \cdot 1.91 \cdot \bar{V}^3 = \frac{1}{2}1.224 \cdot 1.91 \cdot 20^3 = 9351 \ \ W / m^2$$

when temperature is -5 ^oC

$$\rho_{10m,15^oC} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15-5}\exp(-0.0342\frac{10}{273.15-5}) = 1.315 \ kg / m^3$$

$$E(\underset{\sim}{p}) = E(\frac{1}{2}\rho \underset{\sim}{V}^3) = \frac{1}{2}1.315 \cdot E(\underset{\sim}{V}^3) \approx \frac{1}{2}1.315 \cdot 1.91 \cdot \bar{V}^3 = \frac{1}{2}1.315 \cdot 1.91 \cdot 20^3 = 10047 \ \ W / m^2$$

(*ii*).

when temperature is 15 ^oC

$$annual\ energy = 8760 \cdot \eta E(\underset{\sim}{P}) = 8760 \cdot 0.3 \cdot E(\underset{\sim}{p}) \cdot A = 8760 \cdot 0.3 \cdot 9351 \cdot \pi(\frac{60}{2})^2 = 69447 \quad MWh$$

when temperature is -5 ^oC

$$annual\ energy = 8760 \cdot \eta E(\underset{\sim}{P}) = 8760 \cdot 0.3 \cdot E(\underset{\sim}{p}) \cdot A = 8760 \cdot 0.3 \cdot 10047 \cdot \pi(\frac{60}{2})^2 = 74616 \quad MWh$$

Problem b.

D, see 7.5.1

Problem c.

Referred to topic7 slides:

$$E\left(\underset{\sim}{V}^3\right) = \int_0^\infty f_{\underset{\sim}{V}}\left(v\right)dv = \int_0^\infty v^3 \frac{\pi v}{2\left(\bar{v}\right)^2} e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]} dv = \frac{6}{\pi}\left(\bar{v}\right)^3 \approx 1.91\left(\bar{v}\right)^3$$

$$\frac{E\left(P(\underset{\sim}{V}^3)\right)}{P(\bar{v})} = \frac{\int_0^\infty \frac{1}{2}\rho A v^3 \cdot f_{\underset{\sim}{V}}\left(v\right)dv}{\frac{1}{2}\rho A \bar{v}^3} = \frac{\int_0^\infty v^3 f_{\underset{\sim}{V}}\left(v\right)dv}{\bar{v}^3} \approx 1.91$$

If we consider the Rayleigh probability density function as the wind speed distribution function,the average wind power is 1.91 times of the compute wind power using average wind speed